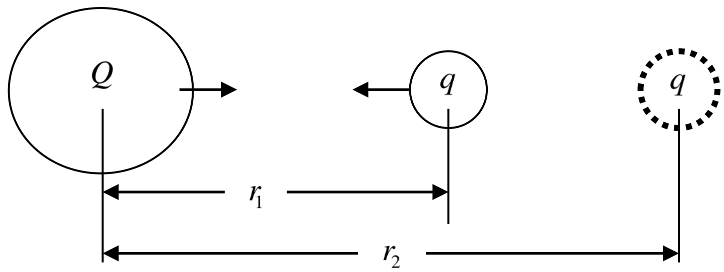
Electric Potential Energy

In the diagram below, the two charges, Q and q, are moved from a separation of r_1 to a separation of r_2, by a force that just overcomes the electric force between them at every point along the path. They are at rest at both positions.



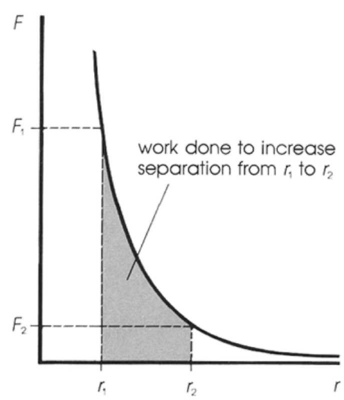
The electric force between the two masses, at any separation distance r, is given by

$$F_E = \frac{kQq}{r^2}$$

To increase the separation of the two masses from r_1 to r_2 requires work to be done to overcome their force of attraction. As a result, the electric potential energy of the system increases.

$$W = \Delta PE_E$$

The amount of work required to increase the separation of the two masses can be calculated from a graph of Force vs. Separation.



This calculation, which involves calculus, results in the following expression

$$\begin{aligned}\Delta PE_E &= PE_E' - PE_E \\ &= \left(\frac{kQq}{r'}\right) - \left(\frac{kQq}{r}\right)\end{aligned}$$

From this expression, we may conclude that

$$PE_E = \frac{kQq}{r}$$

Note:

- If Q and q are opposite charges, they attract, and the electric potential energy should have a negative value. Energy is stored by moving them farther apart.
- If Q and q are similar charges, they repel, and the electric potential energy should have a positive value. Energy is stored by moving them closer together.

Electric Potential

The **electric potential** is defined to be the electric potential energy per unit positive charge. At a distance r from a spherical point charge Q, the electric potential is given by

$$V = \frac{PE_E}{q}$$

Or, using the formula for electric potential energy,

$$V = \frac{kQ}{r}$$

Where,

V is the electric potential measured in volts (V) ($1\ V = 1\ J / C$)

q is the charge on a positive test charge in the field

The electric potential at any point *P* represents the amount of work per unit charge required to bring a charge from infinity to *P*.

It is more common, however, to think of moving a charge from one point to another in the field. In this case, we are concerned with the difference in electric potential between these points, given by

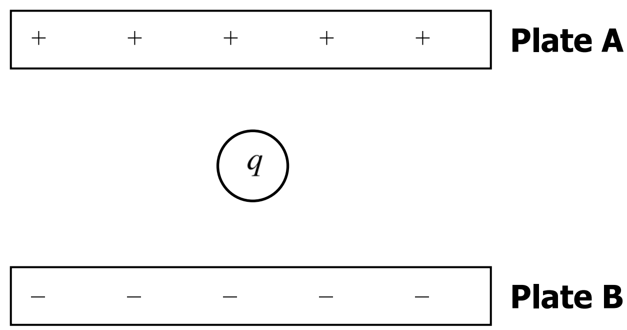
$$\Delta V = \frac{\Delta PE_E}{q}$$

ΔV is referred to as the **electric potential difference** or **voltage**. The potential difference always decreases in the direction of the electric field, and increases in the opposite direction.

Example 1
Calculate the electric potential a distance of $0.4\ m$ from a spherical point charge of $+6.4\times10^{-6}\ C$.

Example 2
How much work must be done to increase the potential of a charge of $3.0\times10^{-7}\ C$ by $120\ V$?

Electric Field between Parallel Plates



The electric field between the plates is constant and is defined as:

$$E = \frac{F_E}{q}$$

The increase in electric potential of the charge q, in moving from plate B to plate A, is equal to the work done in moving from B to A. This can be expressed as

$$W = F \cdot d \qquad \text{since } F \text{ and } d \text{ are in the same direction}$$

$$W = qEd \qquad \text{since } F = F_e = qE$$

and, since $\Delta PE_E = W$,

$$q\Delta V = qEd$$

$$\boxed{E = \frac{\Delta V}{d}}$$

This equation describes the electric field at any point in the space between two parallel plates with a potential difference (usually a battery) ΔV. The direction of the electric field is from the positive plate to the negative plate, in the direction of decreasing potential.

Example 3
In a uniform electric field, the potential difference between two points $10\ cm$ apart is $80\ V$. Calculate the magnitude of the electric field intensity.

Example 4
The electric field intensity in the region between two parallel plates is $400\ N/C$. If the plates are connected to a battery with a potential difference of $90\ V$, what is the separation of the plates?

Homework
Electric Potential Worksheet

Electric Potential Worksheet

1. The potential at a distance of 25 *cm* from a point charge is $-6.4\times10^{4}\ V$. What is the sign and magnitude of the point charge? ($-1.8\times10^{-6}\ C$)

2. Calculate the electric potential 0.50 *m* away from a $4.5\times10^{-4}\ C$ point charge. ($8.1\times10^{6}\ V$)

3. A $1.0\times10^{-6}\ C$ test charge is 40 *cm* from a $3.2\times10^{-3}\ C$ charged sphere. How much work was required to move it there from a point 100 cm away from the sphere? (43 *J*)

4. It takes $4.2\times10^{-3}\ J$ of work to move $1.2\times10^{-6}\ C$ of charge from point *X* to point *Y* in an electric field. What is the potential difference between *X* and *Y*? ($3.5\times10^{3}\ V$)

5. How much energy is acquired by an electron, whose charge is $1.6\times10^{-19}\ C$, in moving through a potential difference of $2.5\times10^{4}\ V$? ($4.0\times10^{-15}\ J$)

6. How much work must be done to bring two protons from an infinite distance apart to within $1.0\times10^{-15}\ m$ of each other? ($2.3\times10^{-13}\ J$)

7. Calculate the magnitude of the electric field in a parallel plate apparatus whose plates are $5.0\ mm$ apart and have a potential difference of 300 *V* between them. ($6.0\times10^{4}\ N/C$)

8. What is the electric field intensity between two large parallel plates 2.0 *cm* apart, if a potential difference of 450 *V* is maintained between them? ($2.3\times10^{4}\ N/C$)

9. What potential difference would have to be maintained across the plates of a parallel plate apparatus, if they are 1.2 *cm* apart, to create an electric field of intensity $1.5\times10^{4}\ N/C$? ($1.8\times10^{2}\ V$)

10. What potential difference applied between two parallel plates will produce an electric field strength of $2.5\times10^{3}\ N/C$ if the plates are 8.0 *cm* apart? ($2.0\times10^{2}\ V$)

11. How far apart are two parallel plates if a potential difference of 600 *V* produces an electric field intensity of $1.2\times10^{4}\ N/C$ between them? ($5.0\times10^{-2}\ m$)